

Analysis of the Central Cavity of Axisymmetric Forward Extrusion by the Upper Bound Approach

S. Wu and M. Li

In this paper, the use of a kinematically admissible velocity field to predict the presence of a central cavity in the final stage of axisymmetric forward extrusion is advanced, in accordance with the results of Moire experiments. On the basis of the velocity field, the critical condition for central cavity formation is obtained by the upper bound approach. Furthermore, the quantitative relationships between central cavity formation and process parameters (reduction in area, frictional factors on the ram and chamber wall, relative residual thickness of the of the billet) are studied. The results show that (1) the critical relative residual thickness of the billet used for the central cavity formation is affected primarily by the reduction in area and the frictional factors on the ram, and slightly by the frictional factor on the chamber wall; (2) the relative dimensions of the central cavity increase with a decrease in the relative residual thickness of the billet; (3) the growth rate of the central cavity decreases with an increase in the frictional factors on the ram, but is affected by the frictional factors on the chamber wall only slightly. Good correlation is found between the analytical and experimental results.

1. Introduction

AXISYMMETRIC forward extrusion is a basic process in the manufacturing industry and in the metallurgical industry. A central cavity is often observed as a defect in forward extrusion.[1,2] To ensure the quality of the workpiece, the occurrence of this defect during this process must be prevented. It is important, therefore, to predict the critical conditions for defect formation. Avitzur[3] has indicated that the use of the upper bound approach to predict the occurrence of defects during metalforming is a relatively new development. Kudó and Johnson[4,5] explained the cause of central cavity formation by the flow of the rigid triangle in the deforming body. Avitzur[6,7] had partly studied the central cavity occurring during axisymmetric forward extrusion by means of the upper bound approach and analyzed the influence of process parameters and the workhardening effect of working materials on central cavity formation. However, the frictional factor, which is a basic parameter affecting the central cavity formation, has not been studied extensively to date.

In this paper, use of the kinematically admissible velocity field to predict the presence of a central cavity in the final stage of axisymmetric forward extrusion is advanced, in accordance with the results of the Moire experiments.[8] On the basis of the velocity field, the critical condition for central cavity formation is obtained. Furthermore, the influence of process parameters, particularly frictional factors on the ram and on the chamber wall, on central cavity formation is determined. The experimental results of three working materials agree with the analytical results described in this paper.

Appendix	
R_o	Ram radius
T	Height between ram and bottom of chamber
U_o	Ram velocity
R_f	Exit radius of the chamber
Γ_z	i-th boundary of velocity discontinuity
U_i	Velocity component in the i-direction
l	Land length of the chamber
U_f	Exit velocity of the workpiece
R_c	Dimension of the central cavity
r_c/r_o	Relative dimension of the central cavity
σ_c	Effective flow stress
p_{ave}/σ_o	Average relative ram pressure
$\mid \Delta V \mid$	Value of velocity discontinuity
J^*	Total power consumption for plastic deformation
$\dot{W}_i$	Power of internal deformation
$\dot{W}_f$	Power to overcome frictional losses
$\dot{W}_s$	Power to overcome shear losses on boundaries of velocity discontinuity
$\dot{\varepsilon}_{ij}$	Component of strain-rate tensor
ε_f	Reduction in area of the axisymmetric forward extrusion
σ	Flow stress
ε	Logarithmic strain
T_{cri}/R_o	Critical relative residual thickness of the billet
$m_1(m_2,m_3)$	Frictional factor on the ram (chamber wall or land)
V	Volume

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2. Analysis of the Central Cavity during Axisymmetric Forward Extrusion

2.1 *Use of the Kinematically Admissible Velocity Field to Detect the Presence of a Central Cavity*

Assume that a central cavity with dimensions R_c has formed in the final stage of axisymmetric forward extrusion. (See the Appendix for a list of all variables.) In accordance with the experimental results,[8] use of kinematically admissible velocity

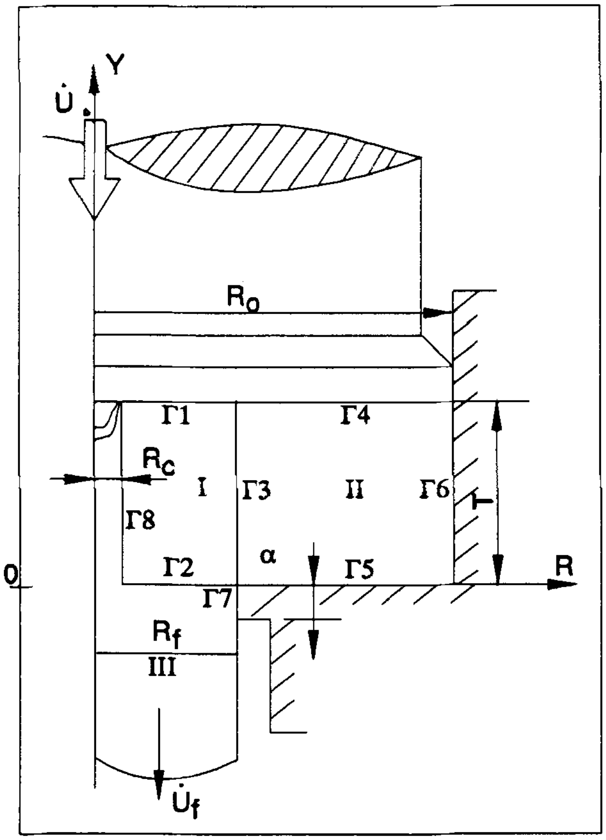

Fig. 1 Flow velocity field used to predict the presence of a central cavity.

field to predict the presence of a central cavity is proposed (Fig. 1). In Fig. 1, Zones I and II are plastic deforming zones, and Zone III is a rigid zone. It can be seen from Fig. 1 that:

$$0 \le R_c \le R_f \qquad [1]$$

Use the cylinder coordinate system (R, θ, Y). On the basis of the volume constancy, the exit velocity of the forward extrusion is

$$\dot{U}_f = \frac{R_o^2 - R_c^2}{R_f^2 - R_c^2}\dot{U}_o$$

Because of the axis symmetry, for every zone

$$\dot{U}_\theta = 0 \qquad [2]$$

In each of the zones, the other velocity components are as follows:

Zone I $\dot{U}_R = \dfrac{R_o^2 - R_f^2}{2T\left(R_f^2 - R_c^2\right)}\left(R - \dfrac{R_c^2}{R_f^2}\right)\dot{U}_o$

$$\dot{U}_Y = \left[\frac{R_o^2 - R_c^2}{R_f^2 - R_c^2} - \frac{R_o^2 - R_f^2}{T\left(R_f^2 - R_c^2\right)}Y\right]\dot{U}_o$$

Zone II $\dot{U}_R = \dfrac{R_o^2 - R^2}{2TR}\dot{U}_o \quad \dot{U}_Y = \dfrac{Y}{T}\dot{U}_o$

Zone III $\dot{U}_R = 0 \quad \dot{U}_Y = \dfrac{R_o^2 - R_c^2}{R_f^2 - R_c^2}\dot{U}_o \qquad [3]$

where R_c is a pseudo-independent variable, which may be obtained by optimization.[11] Equation 3 satisfies geometrical equations, boundary velocity, and volume constancy requirements.[3] The kinematically admissible velocity field to detect the presence of a central cavity (Eq 3′) may also be obtained if we let R_c be equal to zero in Eq 3.

2.2 *Ram Pressure*

Assume that the deformed body is a rigid-perfectly plastic material. In accordance with Eq 3 and Eq 3′, the velocity discontinuities $|\Delta V|$ on the boundaries of the velocity discontinuity and the equivalent strain rates $\left(\sqrt{\tfrac{1}{2}\dot{\varepsilon}_{ij}\,\dot{\varepsilon}_{ij}}\right)$ for every plastic deforming zone may be calculated.[3,5] For the upper bound approach, the general equation used to calculate the power for internal deformation is[3]

$$\dot{W}_i = \sigma_o \int_v \sqrt{\tfrac{1}{2}\dot{\varepsilon}_{ij}\,\dot{\varepsilon}_{ij}}\, dV$$

The general equation to calculate shear power losses is[3]

$$\dot{W}_s = \frac{\sigma_o}{\sqrt{3}} \int_{S_d} |\Delta V|\, dS$$

The general equation for frictional power losses is[3]

$$\dot{W}_f = \frac{m\sigma_o}{\sqrt{3}} \int_{S_f} |\Delta V|\, dS \quad (0 \le m \le 1)$$

Total power consumption for plastic deformation is

$$J^* = \Sigma \dot{W}_i + \Sigma \dot{W}_s + \Sigma \dot{W}_f$$

Based on the power balance principle, the average relative ram pressure for the velocity field with the central cavity is given by

$$p_{\text{ave}}/\sigma_o = \frac{1}{R_o^2}\left\{R_f^2 - R_c^2 + \frac{1}{\sqrt{3}}\left[2R_o^2 + R_o \ln\left(\sqrt{R_o^4 + 3R_f^4} + R_o^2\right) / \sqrt{3}R_f^2 - \sqrt{R_o^4 + 3R_f^4} + \frac{1}{2}R_o \ln 3\right] + \left(\frac{1}{\sqrt{3}} + \frac{m_1}{3}\right)\right.$$

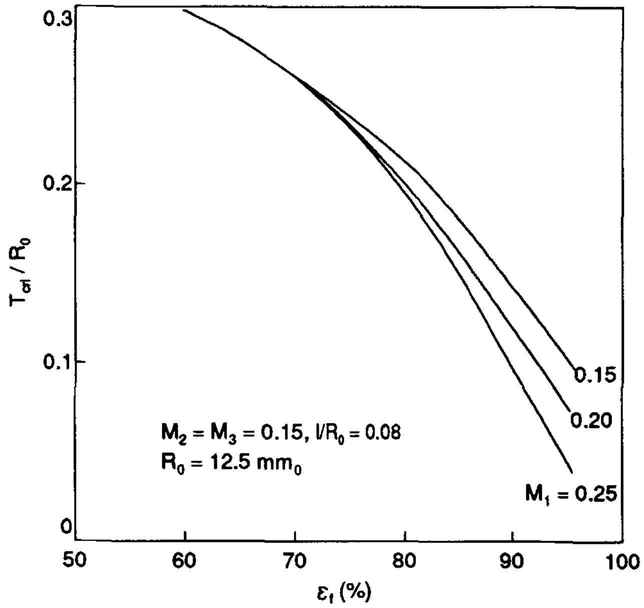

Fig. 2 Effects of frictional factors on the ram and the reduction in area on the critical relative residual thickness of the billet.

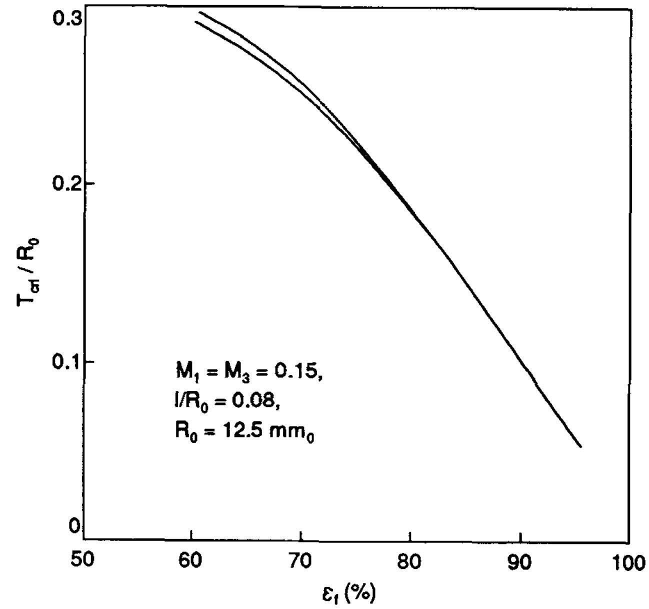

Fig. 3 Effects of frictional factors on the chamber wall and the reduction in area on the critical relative residual thickness of the billet.

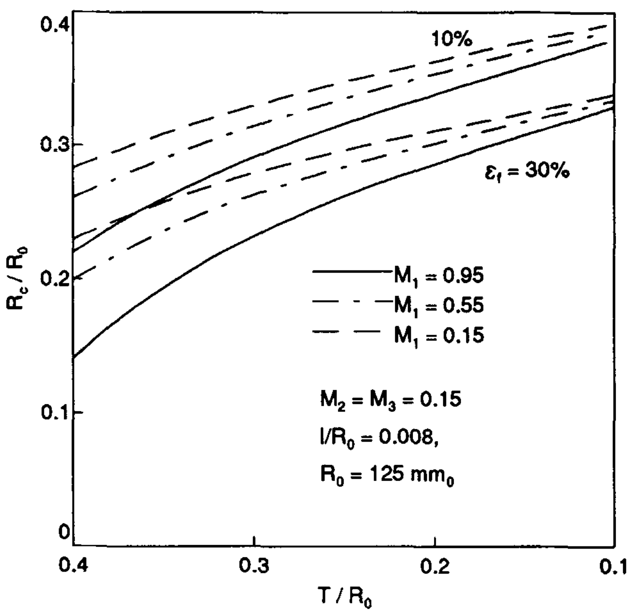

Fig. 4 Effects of frictional factors on the ram and the reduction in area on the relative dimensions of the central cavity.

$$\left[\frac{\left(R_o^2-R_f^2\right)\left(R_f^3-R_c^3\right)}{3T\left(R_f^2-R_c^2\right)}-\frac{R_o^3-R_f^3}{2TR_f}R_c^2\right]+m_2\frac{TR_o}{\sqrt{3}}$$
$$+\frac{R_o^2-R_c^2}{R_f^2-R_c^2}\left[T\left(\frac{R_f}{\sqrt{3}}+R_c\right)+\frac{2m_3}{\sqrt{3}}R_f l\right]$$
$$\left.+\frac{m_1+m_2}{3\sqrt{3}T}\left[2R_o^3-3R_o^2R_f+R_f^3\right]\right\} \qquad [4]$$

Similarly, the average relative ram pressure (Eq 4′) for the flow velocity field without a cavity can be derived. The optimum solution for Eq 4 may be obtained by the optimization.

2.3 *Critical Condition for Central Cavity Formation*

According to the general criterion for macrodefect formation,[3,9,10] the critical condition for central cavity formation in the final stage of axisymmetric forward extrusion is

$$\partial\frac{\left(p_{\text{ave}}/\sigma_o\right)}{\partial R_c}\bigg|\cdot R_c \to 0^+ \le 0 \qquad [5]$$

It is very difficult to solve an equation such as Eq 5. Actually, under a given set of conditions, the critical residual thickness of the billet required for central cavity formation may be determined easily. The reason is as follows: a cavity that occurs during a manufacturing process does so because the flow associated with cavity formation requires less energy than sound flow under the same set of conditions. Comparing the average relative ram pressures for flow in the presence of a central cavity and for sound flow in the final stage, the critical relative residual thickness of the billet required for cavity formation under a given set of conditions is defined.[9,10]

2.4 *Critical Relative Residual Thickness of the Billet Required for the Occurrence of a Central Cavity*

Figures 2 and 3 show the relationships between the central cavity formation and the reduction in area, or the frictional factors. From these figures, it can be seen that the critical relative residual thickness of the billet increases with a decrease in the reduction in area of the forward extrusion. Also, the critical relative residual thickness of the billet decreases with an in-

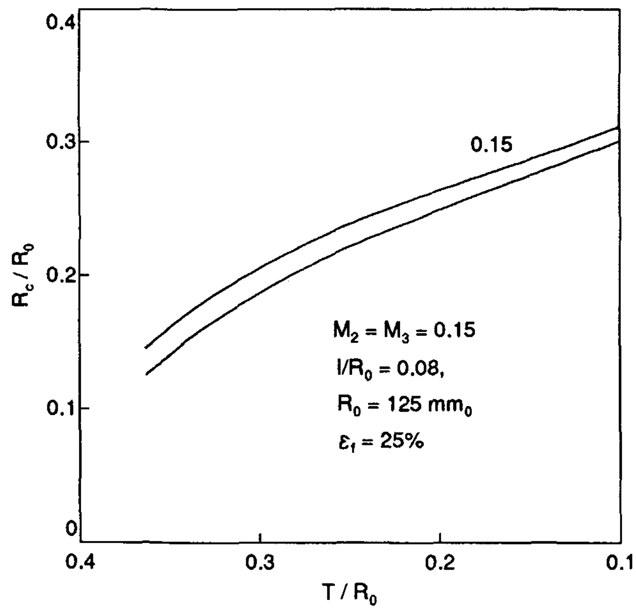

Fig. 5 Effects of frictional factors on the chamber wall and the reduction in area on the relative dimensions of the central cavity.

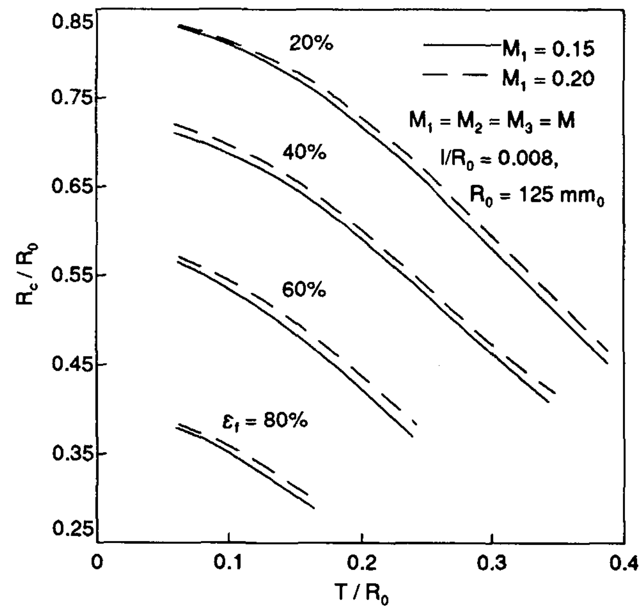

Fig. 6 Relationship between the relative dimensions of the central cavity and the relative residual thickness of the billet for different reductions in area.

crease in the frictional factor on the ram, except that the reduction in area is greater than 70%. The influence of the frictional factors on the chamber wall on central cavity formation is very slight.

2.5 *Relative Dimensions of the Central Cavity*

For a given set of conditions, the relative dimensions of the central cavity are evaluated as shown in Fig. 4 and 5. It can be seen from these figures that the relative dimensions of the central cavity increase with a decrease in the reduction in area, the relative residual thickness of the billet, or the frictional factor on the ram. However, it varies with a change in the frictional factor on the chamber wall only slightly. Figure 6 shows the relationship between the relative dimensions of the central cavity and the relative residual thickness of the billet for different reductions in area under the conditions of general lubrication ($m = 0.10$ to 0.15).

3. Experiments and Verifications

3.1 *Experimental Conditions*

Three working materials were used for the experiments. Specimens of aluminum alloy LY12 (corresponding to ASTM 2024) made from a round bar were annealed at 410 °C for 3 hr and furnace-cooled. Specimens of copper T2 (corresponding to ASTM 102) were annealed at 710 °C for 4 hr and furnace-cooled to 150 °C and then air-cooled to room temperature. Specimens of pure aluminum L4 (corresponding to ASTM 1030) made from a round bar were used to study without any additional treatment. The annealed specimens were immersed in a solution of 30% NaOH at 80 to 100 °C and then in a solution of 50% HNO_3 at room temperature to remove the oxide film formed during annealing.

The method adopted for determining the frictional factor (m) was to carry out a compression test on flat, ring-shaped specimens.[12] The dimensions of the rings for the compression test were 40 mm OD, 20 mm ID, and 13.3 mm in height (6:3:2:). The frictional factors were found to be 0.14 and 0.12, respectively, for LY12 and T2 specimens, which used rape oil as a lubricant, and 0.15 for the L4 specimen, which used mechanical oil as a lubricant.

Tensile tests were carried out at room temperature with a type WE-600kN material test machine. The tensile specimens were machined so as to be 5 mm in diameter and 25 mm in effective length. The experimental results are as follows:

For LY12: $\sigma = 300\,\varepsilon^{0.11}$ [6a]

For T2: $\sigma = 396\,\varepsilon^{0.27}$ [6b]

For L4: $\sigma = 140\,\varepsilon^{0.40}$ [6c]

Experiments for axisymmetric forward extrusion were carried out with a type WE-600kN material test machine. The reduction in area (ε_f) of the axisymmetric forward extrusion was equal to 40, 55, or 70%. The dimensions of the specimens for the extrusion test were 24.9 mm in diameter and 25.0 mm in height, 24.9 mm in diameter and 20.0 mm in height, or 24.9 mm in diameter and 12.5 mm in height.

3.2 *Comparison of Results*

The logarithmic strain for the axisymmetric forward extrusion is calculated as follows:

$$\varepsilon = -\ln(1 - \varepsilon_f) \qquad [7]$$

Table 1 affords a comparison between the calculated and the experimental results in axisymmetric forward extrusion. In

Table 1 Comparison between Calculated and Experimental Results of Central Cavity Formation

Working material	E_f,%	T_o/R_o	T/R_o	m	Computed force, kN Without defect, p'_{ave}	With defect, p_{ave}	Central cavity formation Predicted	Experimental	Relative dimension of defect, R_c/R_o Calculated	Measured
L4R	55	1.6	0.48	0.15	120.4	124.6	No	No	...	...
		1.6	0.28	0.15	120.8	120.0	Central cavity	Central cavity	0.36	0.34
LY12.........	40	1.0	0.37	0.14	197.7	190.6	Central cavity	Central cavity	0.32	0.40
		2.0	0.66	0.14	205.4	216.7	No	No	...	...
		2.0	0.26	0.14	195.8	191.1	Central cavity	Central cavity	0.51	0.55
	55	2.0	0.68	0.14	294.3	296.5	No	No	...	...
T2............	40	1.0	0.64	0.12	256.2	256.6	No	No	...	...
		2.0	0.31	0.12	229.1	227.1	Central cavity	Central cavity	0.43	0.48
	70	2.0	0.08	0.12	648.9	541.6	Central cavity	Central cavity	0.47	0.47

Note: $m_1 = m_2 = m_3 = m$, $l/R_o = 0.08$, $R_o = 12.5$ mm.

consideration of the work-hardening effect, substitute σ calculated from Eq 6 and 7 for σ_o in Eq 4 and 4′. The relative dimensions of the central cavity presented in Table 1 were measured with a tool-microscope.

4. Conclusions

The kinematically admissible velocity field advanced in this article is more suitable for simulating the central cavity formation of the forward extrusion. An increase in the frictional factor on the ram may restrain the growth of the central cavity, but the effect of the frictional factor on the chamber wall on the central cavity is very slight. The relative dimensions of the central cavity increases with a decrease in the relative residual thickness of the billet.

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